COV886 Special Module in Algorithms: Computational Social Choice

Lecture 12

Fairness and Efficiency

April 02, 2022 | Rohit Vaish

Reminder about starting recording

The Model

	A	B	C	D	E
	4	1	2	2	2
	1	0	5	1	1
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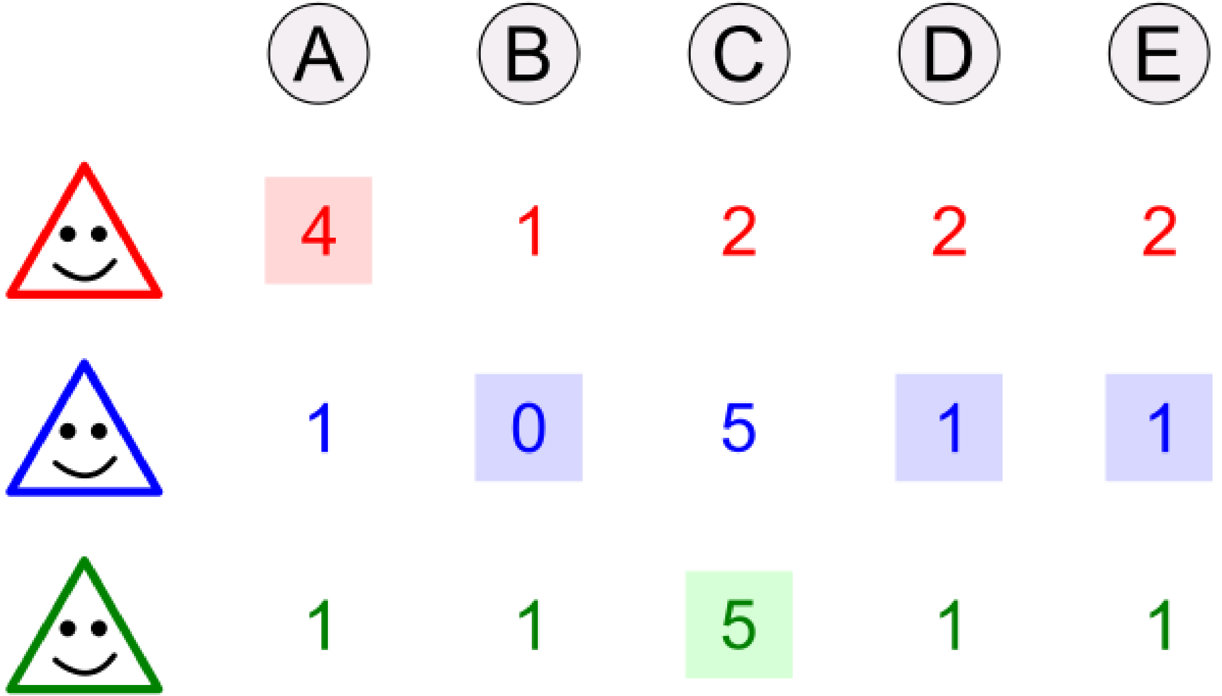
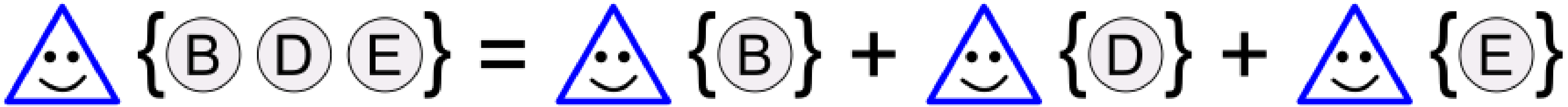


Additive valuations



$= 0+1+1 = 2$

Envy-Freeness Up To One Good [Budish, 2011]

Envy can be eliminated by removing some good in the envied bundle.

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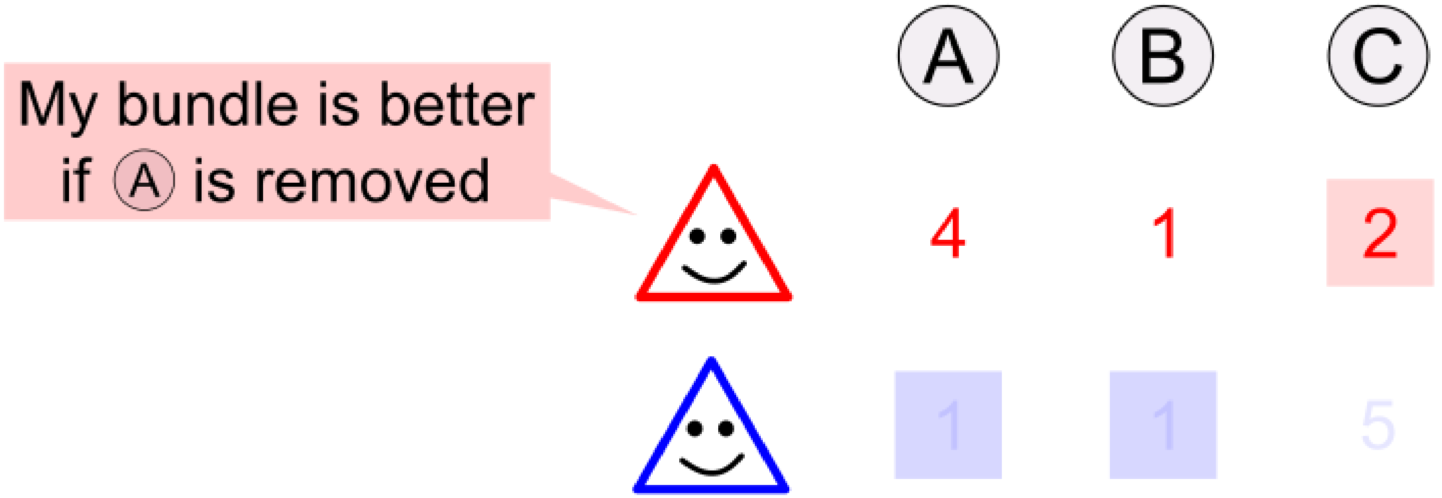

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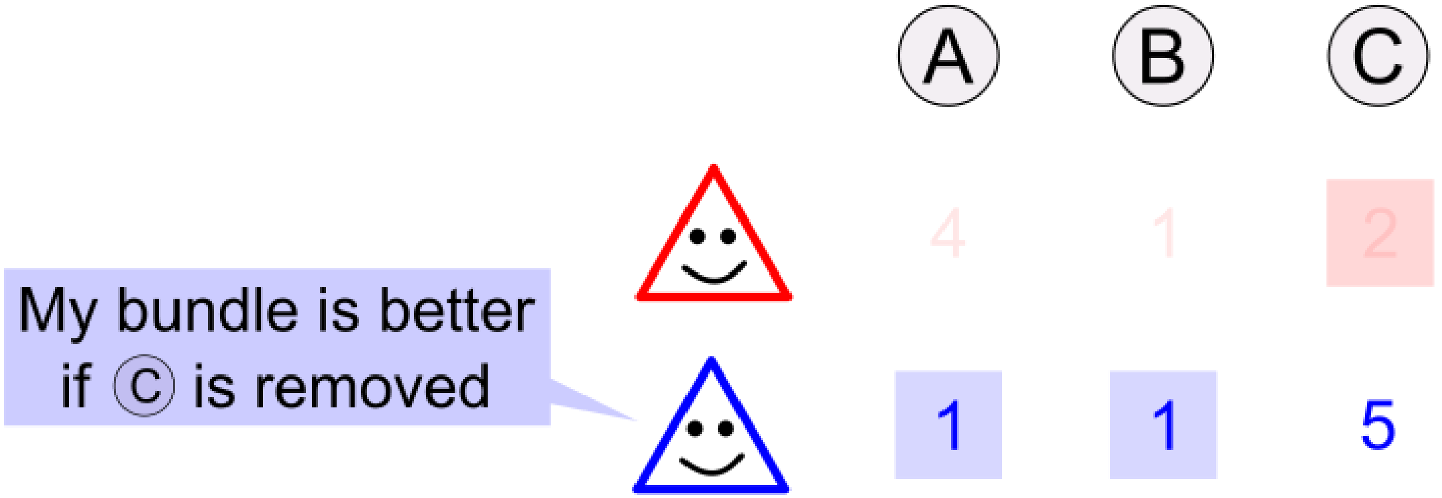

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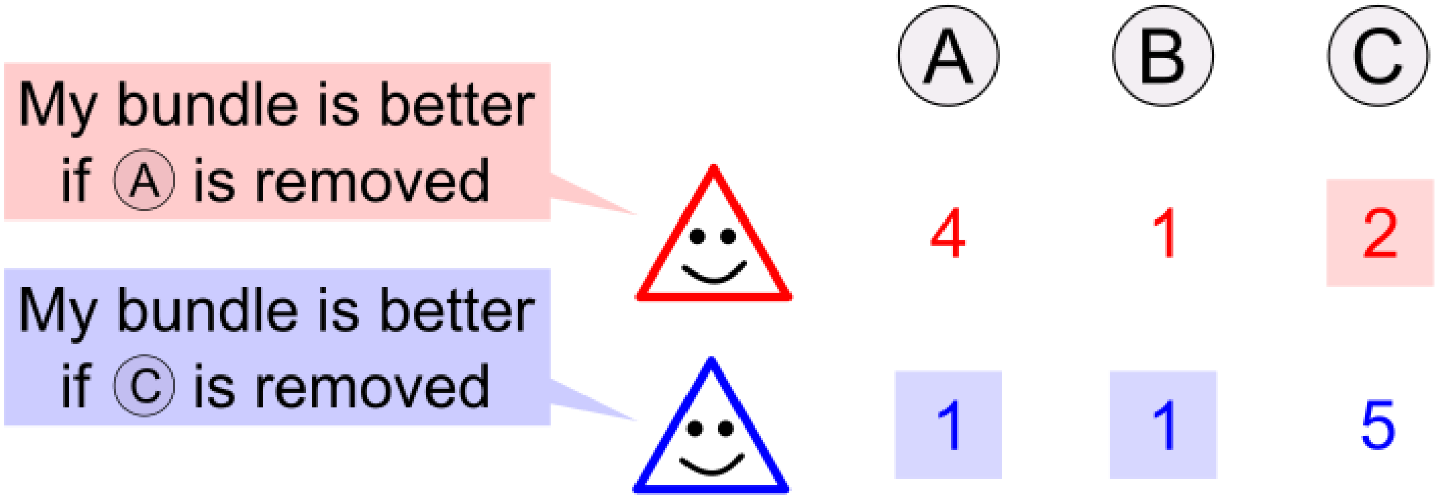

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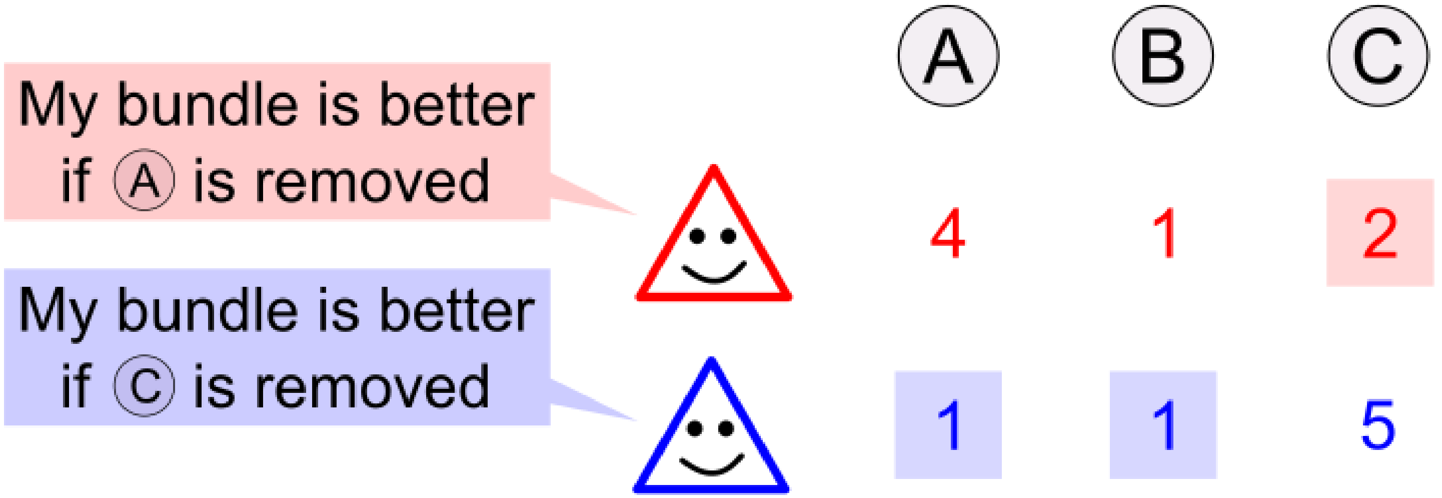

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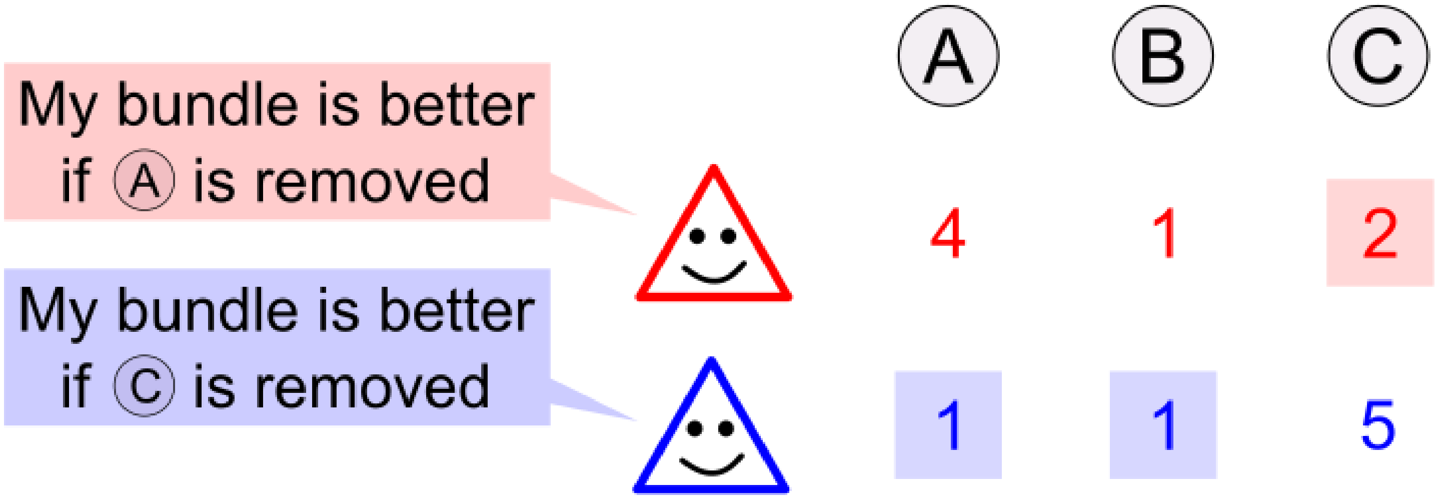

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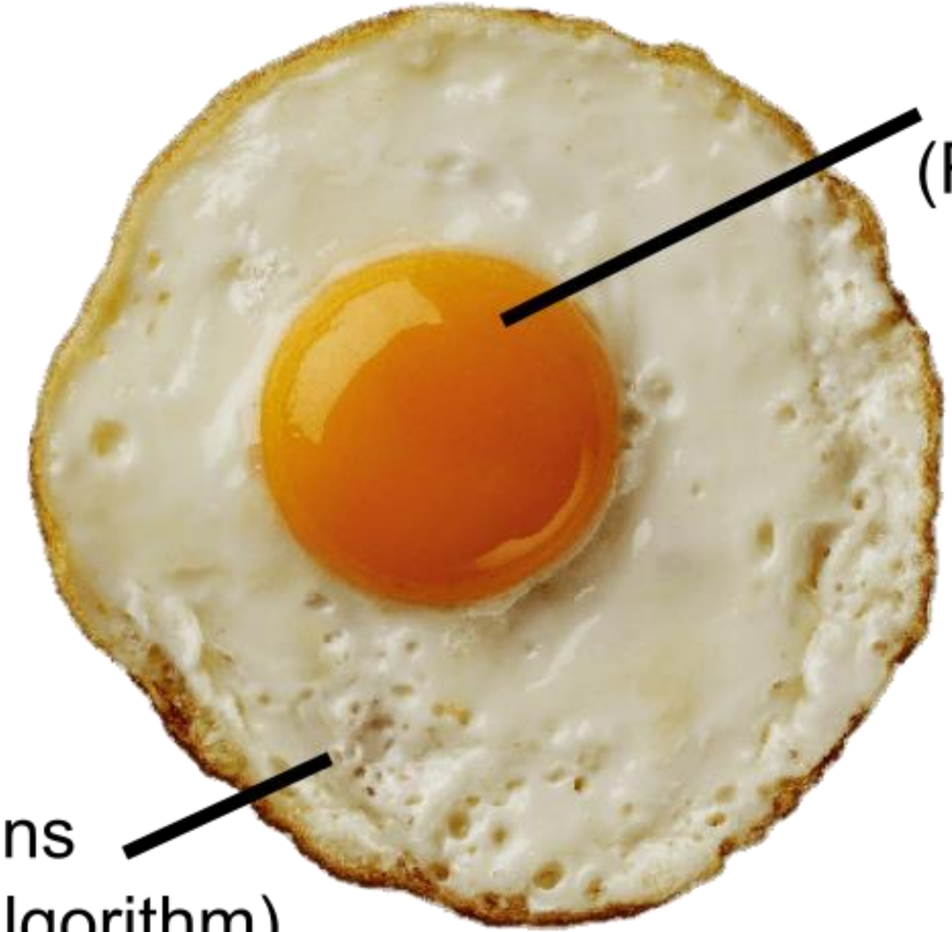
Guaranteed to exist and efficiently computable

Last Time

Algorithms for finding an EF1 allocation



Additive valuations
(Round-robin algorithm)

Monotone valuations
(Envy-cycle elimination algorithm)

WHEN A COMPLETE ALLOCATION

SIMPLY ISN'T ENOUGH

"Obvious" Improvement

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	4	3	1	1	1
	5	2	1	1	1

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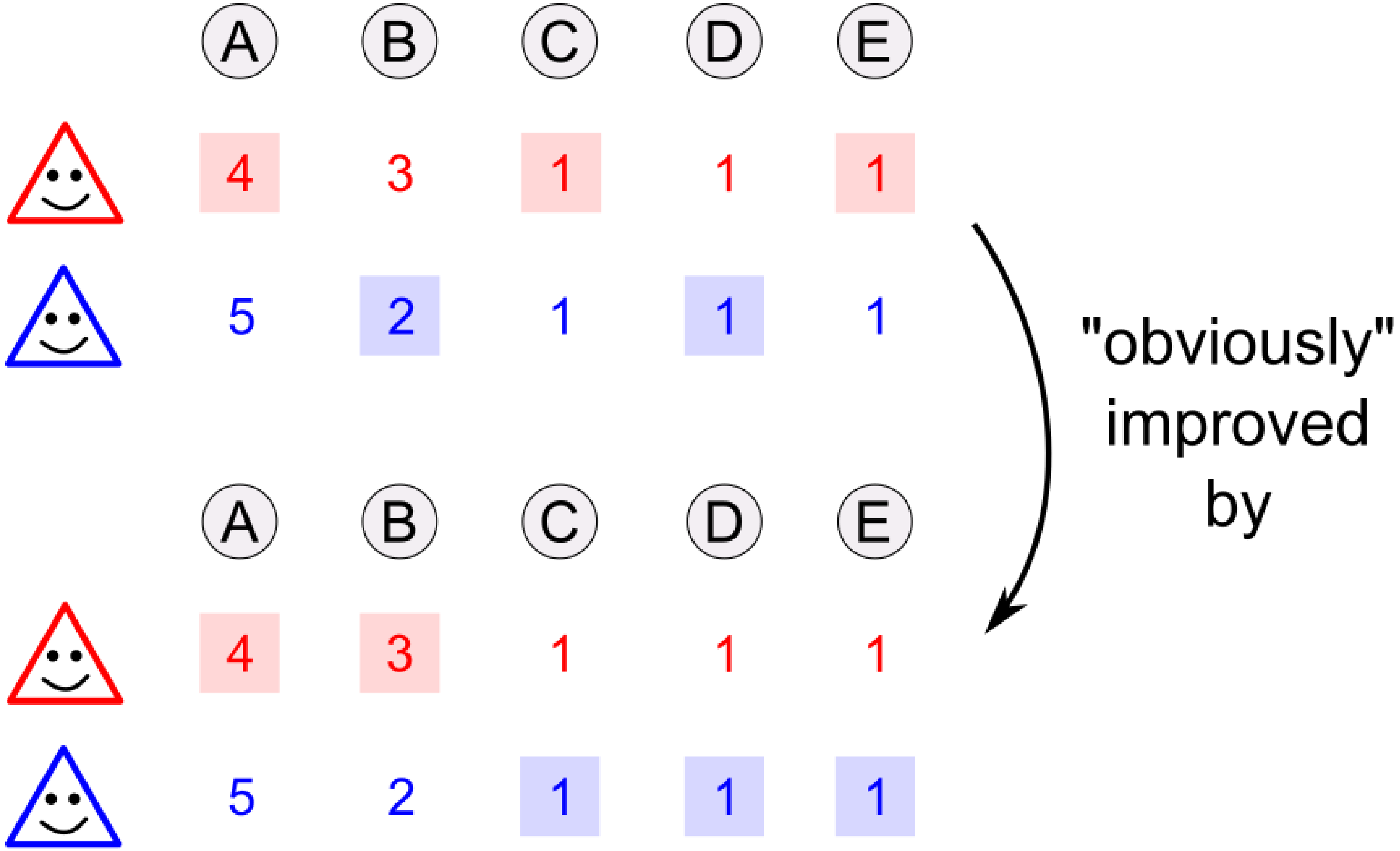

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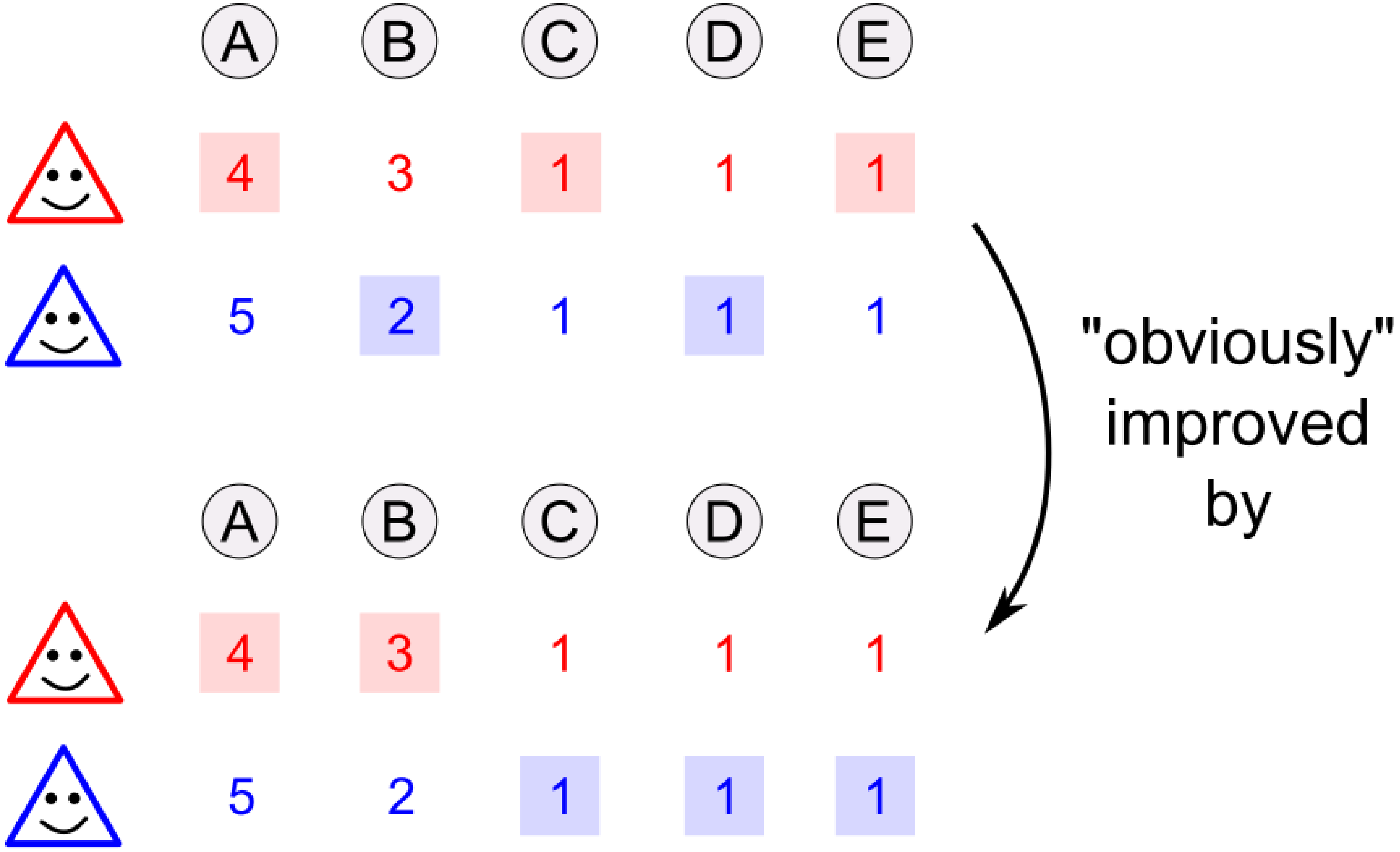

Strictly improving someone without hurting anyone else

Pareto Optimality

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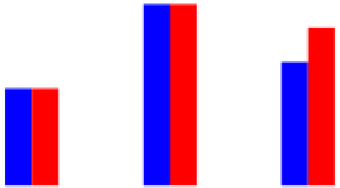
Allocation B *Pareto improves* A if 

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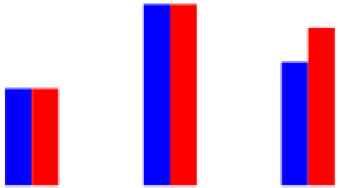

Allocation A is *Pareto optimal* (PO) if no other allocation B Pareto improves it.

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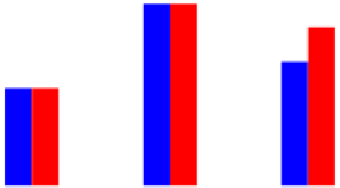
Allocation B *Pareto improves* A if 

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 Guaranteed to exist and efficiently computable

Is EF1 compatible with Pareto optimality?

Round Robin Fails Pareto Optimality

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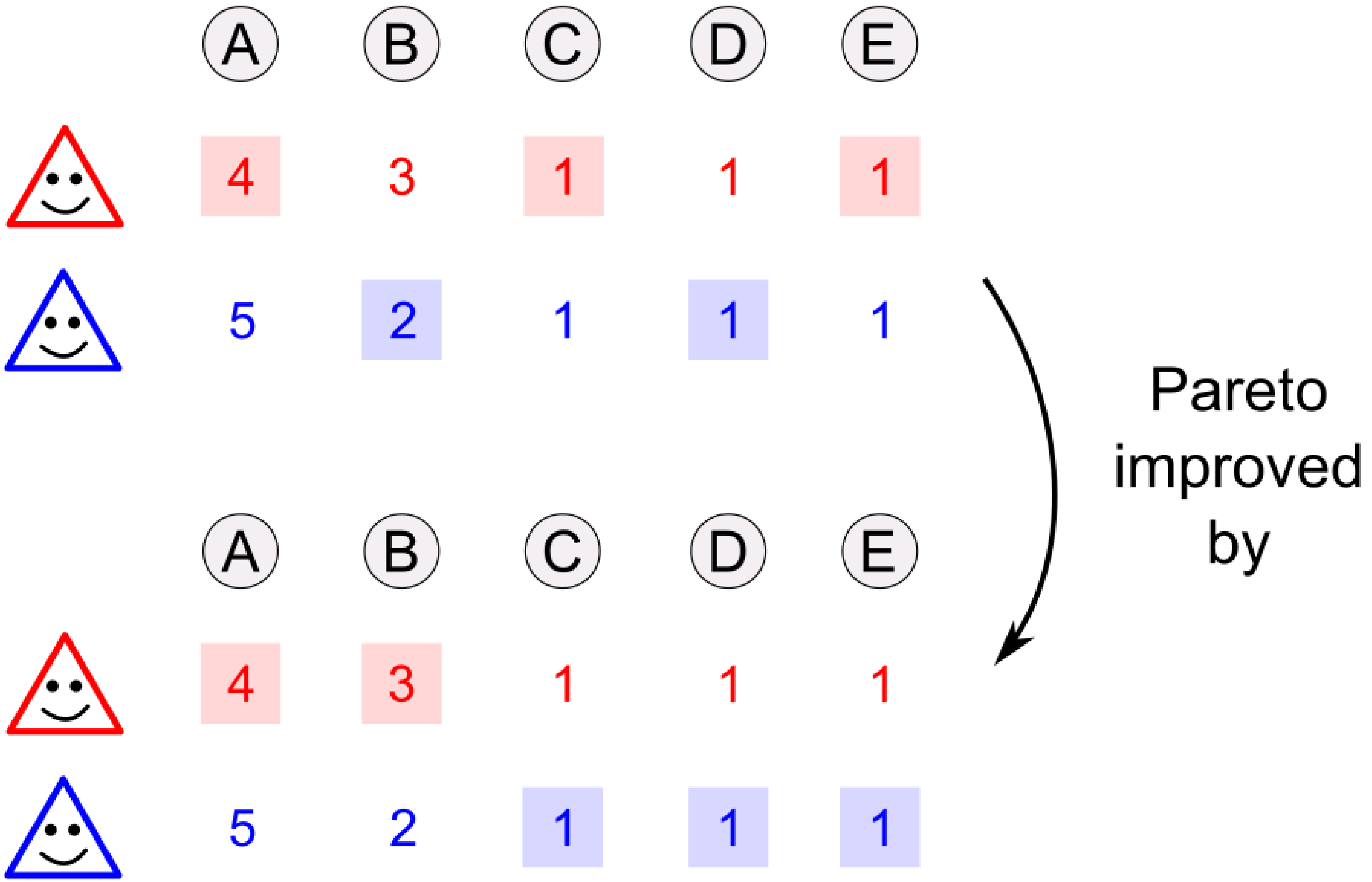

Envy-Cycle Elimination Fails Pareto Optimality

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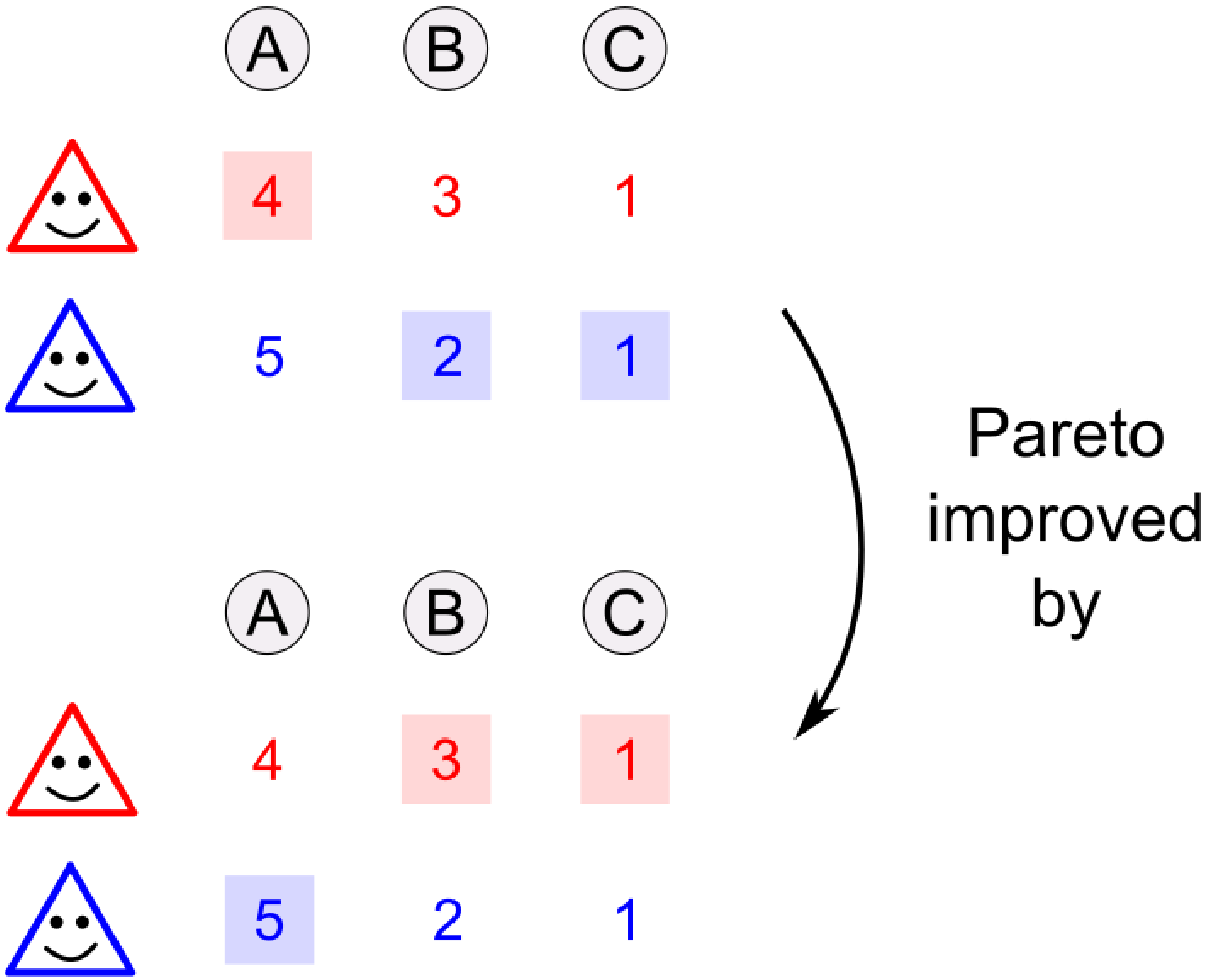

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Start with an EF1 allocation, and repeatedly make Pareto improvements to it.

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Exercise: Pareto improvement can fail to preserve EF1.

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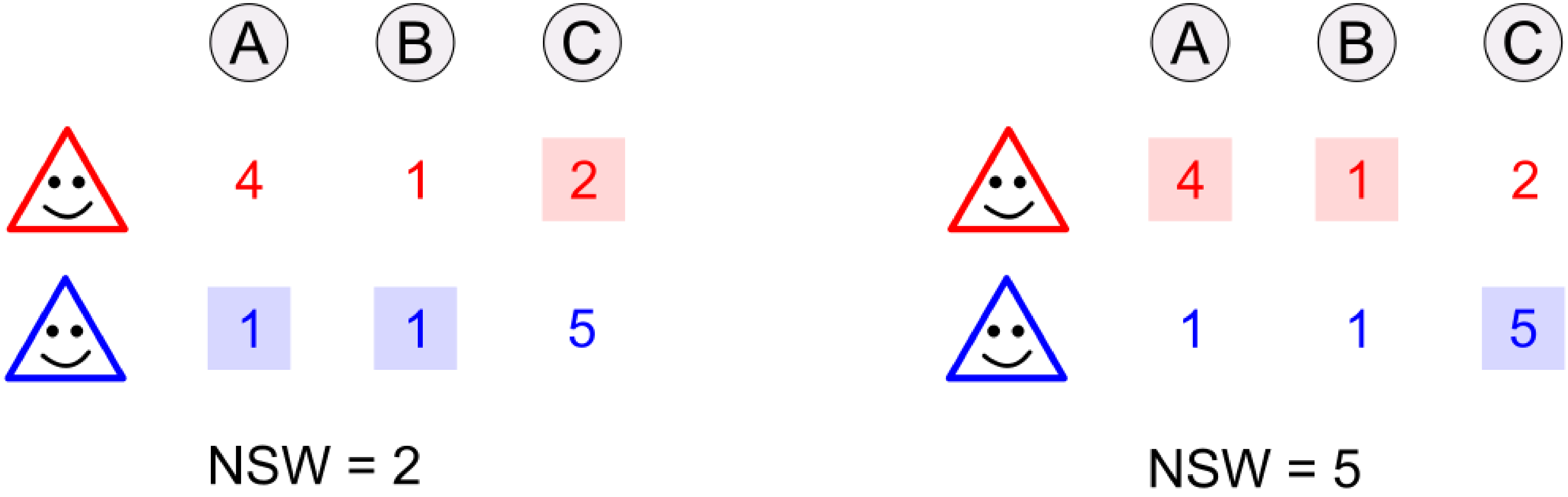

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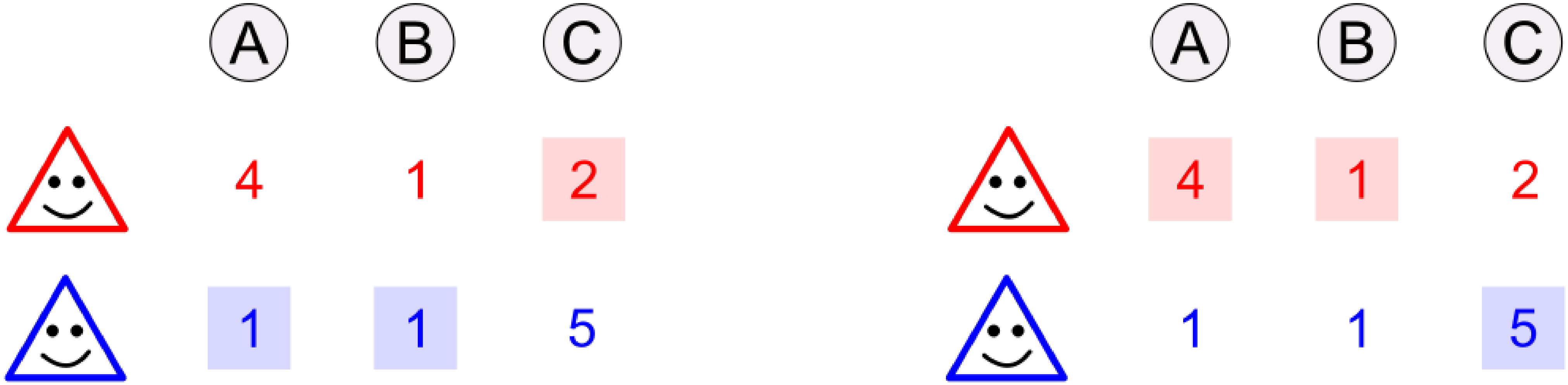

NSW = 2

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A Nash optimal allocation is one that maximizes Nash social welfare.*

*If optimal is 0, then find the largest set of agents who can simulataneously be given positive utility and maximize the geometric mean with respect to only those agents.

[Caragiannis, Kurokawa, Moulin, Procaccia, Shah, and Wang, *TEAC* 2019]

Any Nash optimal allocation satisfies Pareto optimality (PO) and envy-freeness up to one good (EF1).

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Why PO?

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Why PO?

Pareto improvement strictly improves NSW.

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Does such a good g* always exist?

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We will show that transferring g* from agent k to agent i improves NSW.

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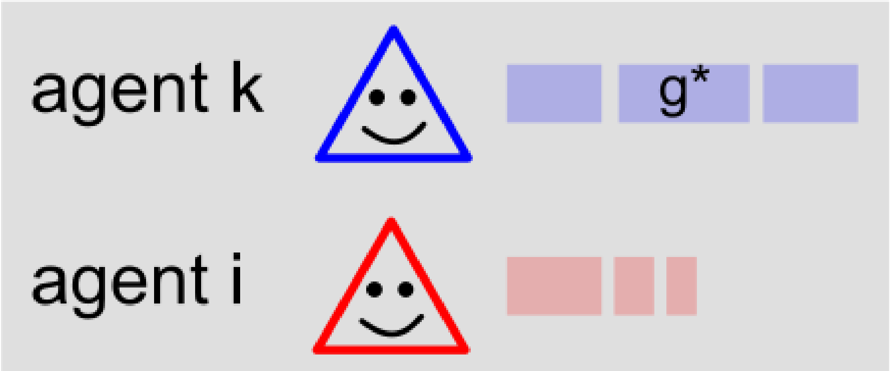

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$$v_i(B_i) = v_i(A_i) + v_i(g^*)$$

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We will show that NSW(B) > NSW(A).

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$$\Leftrightarrow \left(1 - \frac{v_k(g^*)}{v_k(A_k)}\right) \cdot \left(1 + \frac{v_i(g^*)}{v_i(A_i)}\right) > 1$$

[Caragiannis, Kurokawa, Moulin, Procaccia, Shah, and Wang, *TEAC* 2019]

Any Nash optimal allocation satisfies Pareto optimality (PO) and envy-freeness up to one good (EF1).

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← combining these

■

Ok, so an EF1+PO allocation always exists.

But what about computation?

Can a Nash optimal allocation be efficiently computed?

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[Lee, *Info. Proc. Lett.* 2017]

Maximizing Nash social welfare is APX-hard.

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- Even for bounded valuations

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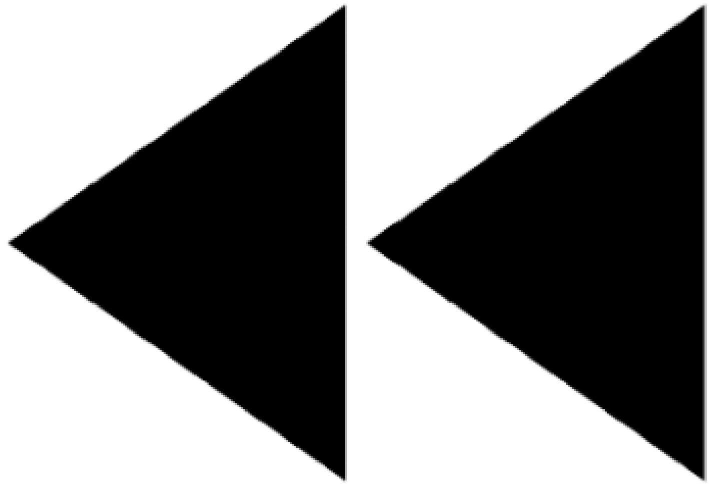
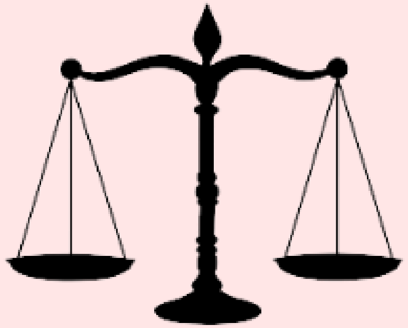
- Running time depends on $v_{i,j}$'s rather than log $v_{i,j}$'s
- Polynomial time for bounded valuations
- A 0.69-approximation to Nash social welfare objective

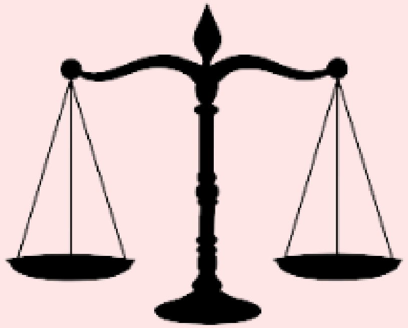



VOTE



[Lec 1+2]

Voting rules:
Good, bad, and dictatorial



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NP-hardness of manipulating
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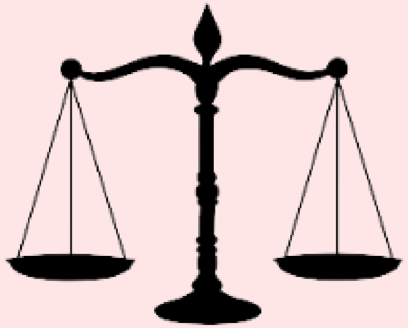

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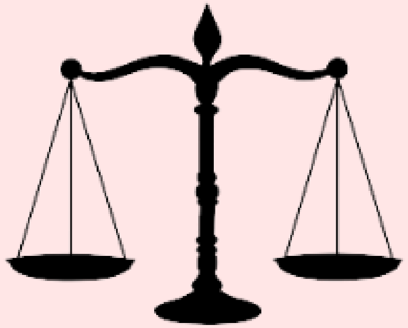
Detecting single-peaked preferences



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Top-trading cycle algorithm

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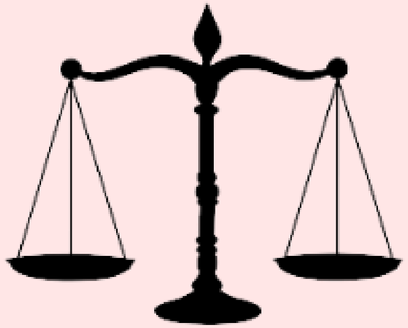
[Lec 7]

Inconspicuous manipulation



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Geometric decomposition via linear programming

[Lec 1+2]

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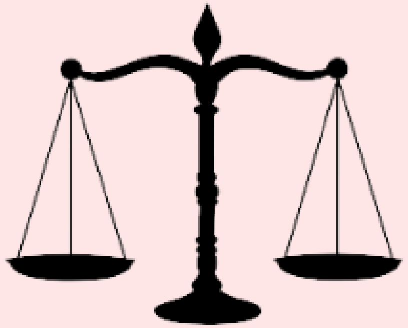

[Lec 9]

Envy-free cake-cutting

[Lec 1+2]

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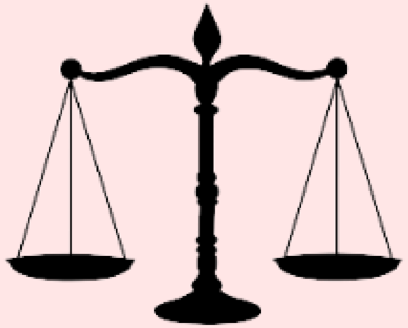

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Rent division via Sperner's lemma

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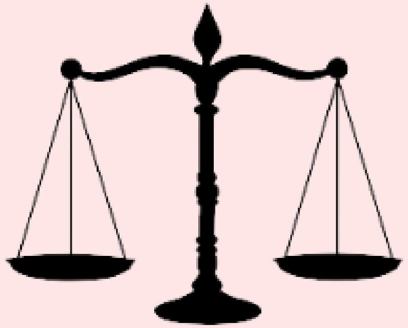

[Lec 9]

Envy-free cake-cutting

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Rent division via Sperner's lemma

[Lec 11+12]

Fair division of the indivisibles

Next Semester

COL866: Special Topics in Algorithms

- A deeper dive into topics in CS + Economics
- Great starting point for a research project or thesis in this area

Reminders

Project presentations in online mode on Apr 03 (Sun).

Project reports due by Apr 10 (Sun).

Thank you!

References

- Maximum Nash Welfare is EF1 and PO.

  Ioannis Caragiannis, David Kurokawa, Hervé Moulin, Ariel D. Procaccia, Nisarg Shah, and Junxing Wang
  “The Unreasonable Fairness of Maximum Nash Welfare”
  ACM Transactions on Economics and Computation, 7(3), 2019 pg 1-32
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- APX-hardness of maximizing Nash social welfare

  Euiwoong Lee
  “*APX-Hardness of Maximizing Nash Social Welfare with Indivisible Items*”
  Information Processing Letters, 122, 2017 pg 17-20
  https://www.sciencedirect.com/science/article/pii/S0020019017300212

References

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  Siddharth Barman, Sanath Kumar Krishnamurthy, and Rohit Vaish
  “Finding Fair and Efficient Allocations”
  EC 2018, pg 557-574
  https://dl.acm.org/doi/10.1145/3219166.3219176